

A Predictive Yukawa Unified SO(10) Model: Higgs and Sparticle Masses

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Abstract

We revisit a class of supersymmetric SO(10) models with t - b - τ Yukawa coupling unification condition, with emphasis on the prediction of the Higgs mass. We discuss qualitative features in this model that lead to a Higgs mass prediction close to 125 GeV. We show this with two distinct computing packages, Isajet and SuSpect, and also show that they yield similar global features in the parameter space of this model. We find that t - b - τ Yukawa coupling unification prefers values of the CP-odd Higgs mass m_A to be around 600 GeV, with all colored sparticle masses above 3 TeV. We also briefly discuss prospects for testing this scenario with the ongoing and planned direct dark matter detection experiments. In this class of models with t - b - τ Yukawa unification, the neutralino dark matter particle is heavy ($m_{\tilde{\chi}_1^0} \gtrsim 400$ GeV), which coannihilates with a stau to yield the correct relic abundance.

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1 Introduction

Despite no signals from direct Supersymmetry (SUSY) searches at the LHC, a 125 GeV Higgs boson is a strong indication that a SUSY signal might be imminent. SUSY still remains at the forefront of beyond the standard model physics scenarios due to several reasons. In addition to solving the gauge hierarchy problem and providing a dark matter candidate, it also leads to unification of the gauge couplings. In the SUSY case, t - b - τ Yukawa unification (YU) [1] can also be accommodated in contrast to its non-SUSY version. Both these observations hint at an underlying grand unified structure like SO(10) which may be supersymmetric. The implications of t - b - τ YU has been extensively explored over the years [2, 3].

The discovery of the Higgs boson was announced by the ATLAS and the CMS collaborations using the combined 7 TeV and 8 TeV data, with ATLAS observing a 5.0σ signal at $126.0 \pm 0.4(\text{stat}) \pm 0.4(\text{syst})$ GeV [4], and CMS a 5.0σ signal at $125.3 \pm 0.4(\text{stat}) \pm 0.5(\text{syst})$ GeV [5]. This observation was also confirmed at the de Moriond 2013 conference. Spurred by these exciting results, we revisit a previous analysis done in ref. [6] with special emphasis on the prediction of the Higgs mass in a class of SO(10) models with t - b - τ YU condition. It was shown in [6] that with non-universal soft supersymmetry breaking (SSB) gaugino masses, which can be derived in the framework of SO(10) GUT, and with universal SSB scalar Higgs doublet masses at M_{GUT} ($M_{H_u}^2 = M_{H_d}^2$), the mass of CP-even SM-like Higgs boson can be predicted with t - b - τ YU case. In this paper we shed light on the qualitative features that lead to consistency of t - b - τ YU with a 125 GeV Higgs. Furthermore, we employ two different computing packages, namely Isajet 7.84 and SuSpect 2.41, and show that they agree very well with the 125 GeV Higgs mass prediction for t - b - τ YU. Good agreement is found between these two programs over the entire parameter space, thereby rendering our conclusions more robust.

Another motivation to revisit the analysis presented in ref. [6] is the recent discovery of $B_s \rightarrow \mu^+\mu^-$ decay by the LHCb collaboration [7]. The branching fraction $BF(B_s \rightarrow \mu^+\mu^-) = 3.2_{-1.2}^{+1.5} \times 10^{-9}$ is in accord with the SM prediction of $(3.2 \pm 0.2) \times 10^{-9}$ [8]. In SUSY models, this flavor-changing decay receives contributions from the exchange of the pseudoscalar Higgs boson A [9], which is proportional to $(\tan\beta)^6/m_A^4$. Since $\tan\beta \approx 47$ was predicted in ref. [6], it is interesting to see how the parameter space is impacted with the $B_s \rightarrow \mu^+\mu^-$ discovery.

We also intend to highlight another interesting feature of this YU model, namely that in addition to the prediction of a CP even Higgs boson mass of around 125 GeV, the model also prefers the CP odd Higgs boson mass of around 600 GeV. This prediction can hopefully be tested at the LHC in the near future [10]. The colored sparticle masses, consistent with good (10% or better) t - b - τ YU, lie well above the current mass limits from the LHC, i.e., $m_{\tilde{g}} \gtrsim 1.4$ TeV (for $m_{\tilde{g}} \sim m_{\tilde{q}}$) and $m_{\tilde{g}} \gtrsim 0.9$ TeV (for $m_{\tilde{g}} \ll m_{\tilde{q}}$) [11, 12].

The outline for the rest of the paper is as follows. In section 2 we briefly describe an SO(10) GUT model which can yield realistic fermion masses and mixings compatible with t - b - τ YU. In Section 3 we present the parameter space that we scan over, and describe how the MSSM gaugino mass relations can be obtained at M_{GUT} . In Section 4 we summarize the scanning procedure and the experimental constraints applied in our analysis. In section 5 we discuss how SUSY threshold corrections, which are necessary to obtain t - b - τ YU and radiative electroweak symmetry breaking (REWSB), determine the CP-even Higgs boson mass. In section 6 we compare the results from the two packages, Isajet and SuSpect. In section 7 we briefly discuss the little hierarchy problem in the presence of t - b - τ YU, and our conclusions are presented in Section 8.

2 SO(10) GUT with t - b - τ Yukawa unification

One of the main motivations of SO(10) GUT, in addition to gauge coupling unification, is matter unification. The spinor representation of SO(10) unifies all matter fermions of a given family in a single multiplet (16_i), which also contains the right handed neutrino (ν_R) that helps to generate light neutrino masses via the see-saw mechanism [13]. The right handed neutrino can also naturally account for the baryon asymmetry of the Universe via leptogenesis [14]. Another virtue of SO(10) is that, in principle, the two MSSM Higgs doublets can be accommodated in a single ten dimensional representation, which then yields the following Yukawa couplings

$$Y_{ij} \ 16_i \ 16_j \ 10_H. \quad (1)$$

Here $i, j = 1, 2, 3$ stand for family indices and the SO(10) indices have been omitted for simplicity. For the third generation quarks and leptons, the interaction in Eq.(1) yields the following Yukawa coupling unification condition at the GUT scale [1]

$$Y_t = Y_b = Y_\tau = Y_{\nu_\tau}. \quad (2)$$

It is interesting to note that in the gravity mediation SUSY breaking scenario [15], t - b - τ YU condition leads to LHC testable sparticle spectrum [2, 3] and even predicts a 125 GeV light CP-even Higgs boson mass [6]. On the other hand, it is well known that the interaction in Eq.(1) leads to a naive SO(10) relation: $N = U \propto D = L$, where U , D , N and L denote the Dirac mass matrices for up and down quarks, neutrinos and charge leptons respectively. $U \propto D$ would imply vanishing quark flavor (Cabibbo-Kobayashi-Maskawa (CKM)) mixing [16], and $m_c^0/m_t^0 = m_s^0/m_b^0$ which significantly contradicts with experimental observations. The superscript zero refers to the parameters evaluated at the M_{GUT} . $D = L^T$, which is a naive SU(5) relation, would imply $m_s^0 = m_\mu^0$ and $m_d^0 = m_e^0$ and this relation also strongly disagrees with

the measurements. So, it is obvious the interaction in Eq.(1) should be modified in order to accommodate the observed pattern of quarks and mixing.

There are two main approaches to avoid these problems and obtain realistic fermion masses and mixings. One way is to extend the Higgs sector and assume that the SM Higgs doublet fields are superposition of fields from the different SO(10) representations [17]. Another way is to introduce additional vector-like matter multiplets at the GUT scale [18] which mix in a nontrivial way with fermions in the 16 dimensional representation. It is equivalent to introducing non-renormalizable couplings which involve a non-singlet SO(10) field that develops a VEV [19]. Both these cases, however, allow t - b - τ YU to be completely destroyed or partially destroyed to yield b - τ YU. Therefore, in order to maintain t - b - τ YU the following two conditions should be satisfied:

- Third generation charged fermions obtain their masses only from Eq.(1).
- The MSSM Higgs doublets H_u and H_d reside solely in 10_H .

There exist models where t - b - τ YU is maintained to a very good approximation with realistic fermion masses and mixings [20].

We consider the case in which the MSSM Higgs fields are contained in the 10_H representation and assume non-renormalizable couplings in the superpotential

$$Y_{ij} 16_i 16_j 10 + Y'_{ij} 16_i 16_j \left(\frac{45}{M} \right)^n 10 + f_{ij} \frac{16_i 16_j \overline{16}_H \overline{16}_H}{M}. \quad (3)$$

Here M , a scale associated with the effective non-renormalizable interaction, could plausibly lie somewhere between the unification scale and the reduced Planck mass ($M_P = 2.4 \times 10^{18}$ GeV). In Eq.(3) n is an integer and 45 is an adjoint representation of SO(10). We can have several 45-dimensional fields with VEVs in different directions in the space spanned by the 45 generator of SO(10), as long as the SM gauge group is unbroken. Ref. [19] shows that in order to have the correct naive SO(10) and SU(5) relationships for the first two families we need to have at least two 45 dimensional fields with VEVs in different directions in the SO(10) space. If one of the 45-plet develops a VEV along the $B-L$ direction it can naturally lead to the so-called Georgi-Jarlskog relation [21], $m_s^0 \cong m_\mu^0/3$ and $m_d^0 \cong 3m_e^0$. This will give correct masses after evolution of the RGEs to low scale. The second 45-plet pointing towards I_{3R} , the third component of the right handed isospin group, will break $U \propto D$ and will allow for the correct CKM mixing matrix.

The pair of spinors $16_H + \overline{16}_H$ break the rank of the group from five to four and provides a mass for the right handed neutrinos. The third term in Eq.(3) is responsible for generating the right handed neutrino masses. The adjoint 45-plet completes the breaking of SO(10) to the SM gauge group. In order to keep the

MSSM Higgs field in 10_H , the term $h_i 16_H 16_H 10$ allowed in principle, should not be in the superpotential. We can just assume the h_i coupling to be zero, or it can be forbidden by introducing an additional symmetry. It was shown in [19] that the superpotential in Eq.(3) perfectly describes the observed fermion masses and mixings.

Having the MSSM Higgs fields in the 10_H representation, the doublet-triplet splitting problem can be solved in $SO(10)$ using the missing VEV mechanism [22]. A variety of realistic models based on this mechanism have been constructed [23].

3 Fundamental Parameter Space

It has been pointed out [24] that non-universal MSSM gaugino masses at M_{GUT} can arise from non-singlet F-terms, compatible with the underlying GUT symmetry such as $SU(5)$ and $SO(10)$. The SSB gaugino masses in supergravity [15] can arise, say, from the following dimension five operator:

$$-\frac{F^{ab}}{2M_{\text{P}}}\lambda^a\lambda^b + \text{c.c.} \quad (4)$$

Here λ^a is the two-component gaugino field, F^{ab} denotes the F-component of the field which breaks SUSY, the indices a, b run over the adjoint representation of the gauge group. The resulting gaugino mass matrix is $\langle F^{ab} \rangle / M_{\text{P}}$, where the supersymmetry breaking parameter $\langle F^{ab} \rangle$ transforms as a singlet under the MSSM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. The F^{ab} fields belong to an irreducible representation in the symmetric part of the direct product of the adjoint representation of the unified group. This is a supersymmetric generalization of operators considered a long time ago [25].

In $SO(10)$, for example,

$$(45 \times 45)_S = 1 + 54 + 210 + 770. \quad (5)$$

If F transforms as a 54 or 210 dimensional representation of $SO(10)$ [24], one obtains the following relation among the MSSM gaugino masses at M_{GUT} :

$$M_3 : M_2 : M_1 = 2 : -3 : -1, \quad (6)$$

where M_1, M_2, M_3 denote the gaugino masses of $U(1)$, $SU(2)_L$ and $SU(3)_c$ respectively. The low energy implications of this relation have recently been investigated in [26] without imposing YU. In this paper we consider the case with $\mu > 0$ and non-universal gaugino masses defined in Eq.(6). In order to obtain the correct sign for the desired contribution to $(g-2)_\mu$, we set $M_1 > 0$, $M_2 > 0$ and $M_3 < 0$. Somewhat to our surprise, we find that this class of t - b - τ YU models make a rather sharp prediction

for the lightest SM-like Higgs boson mass [6]. In addition, lower mass bounds on the masses of the squarks and gluino are obtained.

Notice that in general, if F^{ab} transforms non trivially under $SO(10)$, the SSB terms such as the trilinear couplings and scalar mass terms are not necessarily universal at M_{GUT} . However, we can assume, consistent with $SO(10)$ gauge symmetry, that the coefficients associated with terms that violate the $SO(10)$ -invariant form are suitably small, except for the gaugino term in Eq.(6). We also assume that D-term contributions to the SSB terms are much smaller compared with contributions from fields with non-zero auxiliary F-terms.

Employing the boundary condition from Eq.(6), one can define the MSSM gaugino masses at M_{GUT} in terms of the mass parameter $M_{1/2}$:

$$\begin{aligned} M_1 &= M_{1/2} \\ M_2 &= 3M_{1/2} \\ M_3 &= -2M_{1/2}. \end{aligned} \tag{7}$$

Note that M_2 and M_3 have opposite signs which, as we will show, is important in implementing Yukawa coupling unification to a high accuracy. In order to quantify Yukawa coupling unification, we define the quantity $R_{tb\tau}$ as,

$$R_{tb\tau} = \frac{\max(y_t, y_b, y_\tau)}{\min(y_t, y_b, y_\tau)}. \tag{8}$$

We have performed random scans for the following parameter range:

$$\begin{aligned} 0 &\leq m_{16} \leq 10 \text{ TeV} \\ 0 &\leq m_{10} \leq 10 \text{ TeV} \\ 0 &\leq M_{1/2} \leq 5 \text{ TeV} \\ 35 &\leq \tan \beta \leq 55 \\ -3 &\leq A_0/m_{16} \leq 3 \end{aligned} \tag{9}$$

Here m_{16} is the universal SSB mass for MSSM sfermions, m_{10} is the universal SSB mass term for up and down MSSM Higgs masses, $M_{1/2}$ is the gaugino mass parameter, $\tan \beta$ is the ratio of the vacuum expectation values (VEVs) of the two MSSM Higgs doublets, A_0 is the universal SSB trilinear scalar interaction (with corresponding Yukawa coupling factored out). We use the central value $m_t = 173.1 \text{ GeV}$ and 1σ deviation ($m_t = 174.2 \text{ GeV}$) for top quark in our analysis [27]. We choose a $+1\sigma$ deviation in m_t since it leads to an increase in the Higgs mass and improves the prediction of the Higgs mass in our analysis. Our results however are not too sensitive to one or two sigma variation in the value of m_t [28]. We use $m_b(m_Z) = 2.83 \text{ GeV}$ which is hard-coded into Isajet.

4 Constraints and Scanning Procedure

We employ Isajet 7.84 [29] and SuSpect 2.41 [30] interfaced with Micromegas 2.4 [31] to perform random scans over the fundamental parameter space. Isajet and SuSpect employ full two loop RGEs for the SSB parameters between M_Z and M_{GUT} . The approach employed by both is similar, but there are some important differences that have been previously studied [32, 33]. SuSpect assumes that the full set of MSSM RGEs are valid between M_Z and M_{GUT} and uses what is referred to as the ‘common scale approach’ in [33]. In this approach the \overline{DR} parameters are extracted at a common scale $M_{EWSB} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. Isajet, on the other hand, uses the ‘step-beta function approach’ which means that it employs one-loop step beta functions for gauge and Yukawa couplings. These two approaches yield very similar results for most of the SUSY parameter space. There are some regions where the discrepancies get magnified [32] and which we observe in our results as well. These differences will be discussed in section 6.

An approximate error of around 2 GeV in the estimate of the Higgs mass in Isajet and SuSpect largely arise from theoretical uncertainties in the calculation of the minimum of the scalar potential, and to a lesser extent from experimental uncertainties in the values for m_t and α_s .

An important constraint comes from limits on the cosmological abundance of stable charged particles [35]. This excludes regions in the parameter space where charged SUSY particles become the lightest supersymmetric particle (LSP). We accept only those solutions for which one of the neutralinos is the LSP and saturates the WMAP bound on the relic dark matter abundance.

Micromegas is interfaced with Isajet and SuSpect to calculate the relic density and branching ratios $BR(B_s \rightarrow \mu^+ \mu^-)$ and $BR(b \rightarrow s \gamma)$. With these codes we implement the following random scanning procedure: A uniform and logarithmic distribution of random points is first generated in the parameter space given in Eq. (9). The function RNORMX [36] is then employed to generate a gaussian distribution around each point in the parameter space. The data points collected all satisfy the requirement of radiative electroweak symmetry breaking (REWSB), with the neutralino in each case being the LSP. After collecting the data, we impose the mass bounds on all the particles [35] and use the IsaTools package [37] to implement the various phenomenological constraints. We successively apply the following experimental constraints on the data that we acquire from SuSpect and Isajet:

$$\begin{aligned}
 0.8 \times 10^{-9} &\leq BR(B_s \rightarrow \mu^+ \mu^-) &\leq 6.2 \times 10^{-9} \quad (2\sigma) & \quad [38] \\
 2.99 \times 10^{-4} &\leq BR(b \rightarrow s \gamma) &\leq 3.87 \times 10^{-4} \quad (2\sigma) & \quad [39] \\
 0.15 &\leq \frac{BR(B_u \rightarrow \tau \nu_\tau)_{\text{MSSM}}}{BR(B_u \rightarrow \tau \nu_\tau)_{\text{SM}}} &\leq 2.41 \quad (3\sigma) & \quad [39] \\
 0 &\leq \Delta(g-2)_\mu/2 &\leq 55.6 \times 10^{-10} & \quad [40]
 \end{aligned}$$

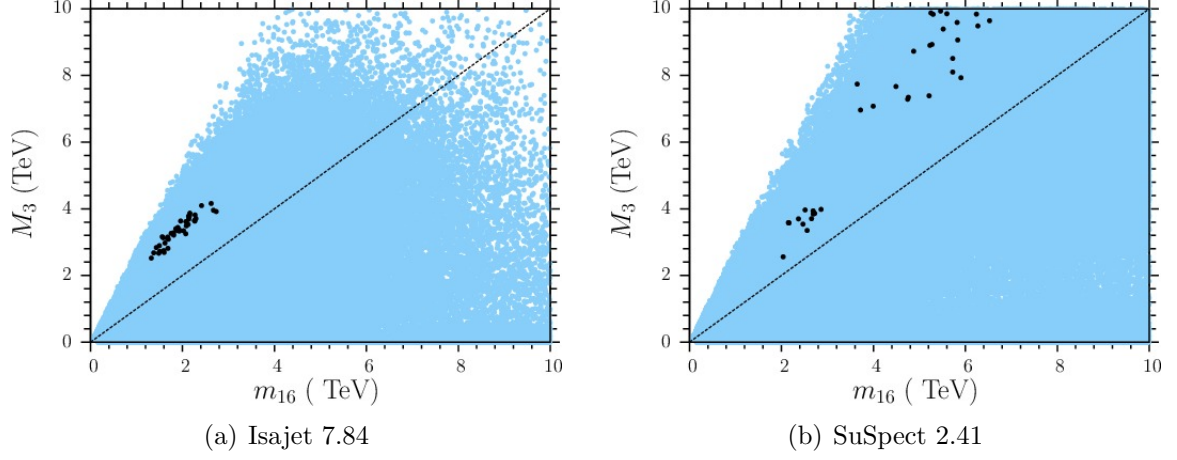


Figure 1: Plot in the $M_3 - m_{16}$ planes. The panel on the left shows data points collected with Isajet 7.84 whereas the panel on the right shows data from SuSpect 2.41. The light blue points are consistent with REWSB and neutralino LSP. For both the panels, black points are subset of the light blue points and satisfy $R_{tb\tau} \leq 1.03$ in panel 1(a) and $R_{tb\tau} \leq 1.05$ in panel 1(b). The unit line is to guide the eye.

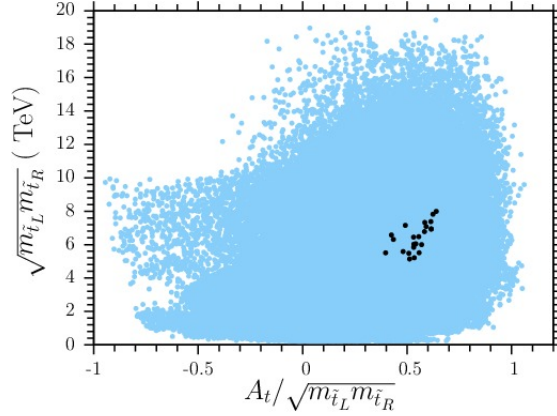


Figure 2: Plot in the $\sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}} - A_t/\sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$ planes. The data points shown are collected using Isajet. Color coding is the same as in Figure 1.

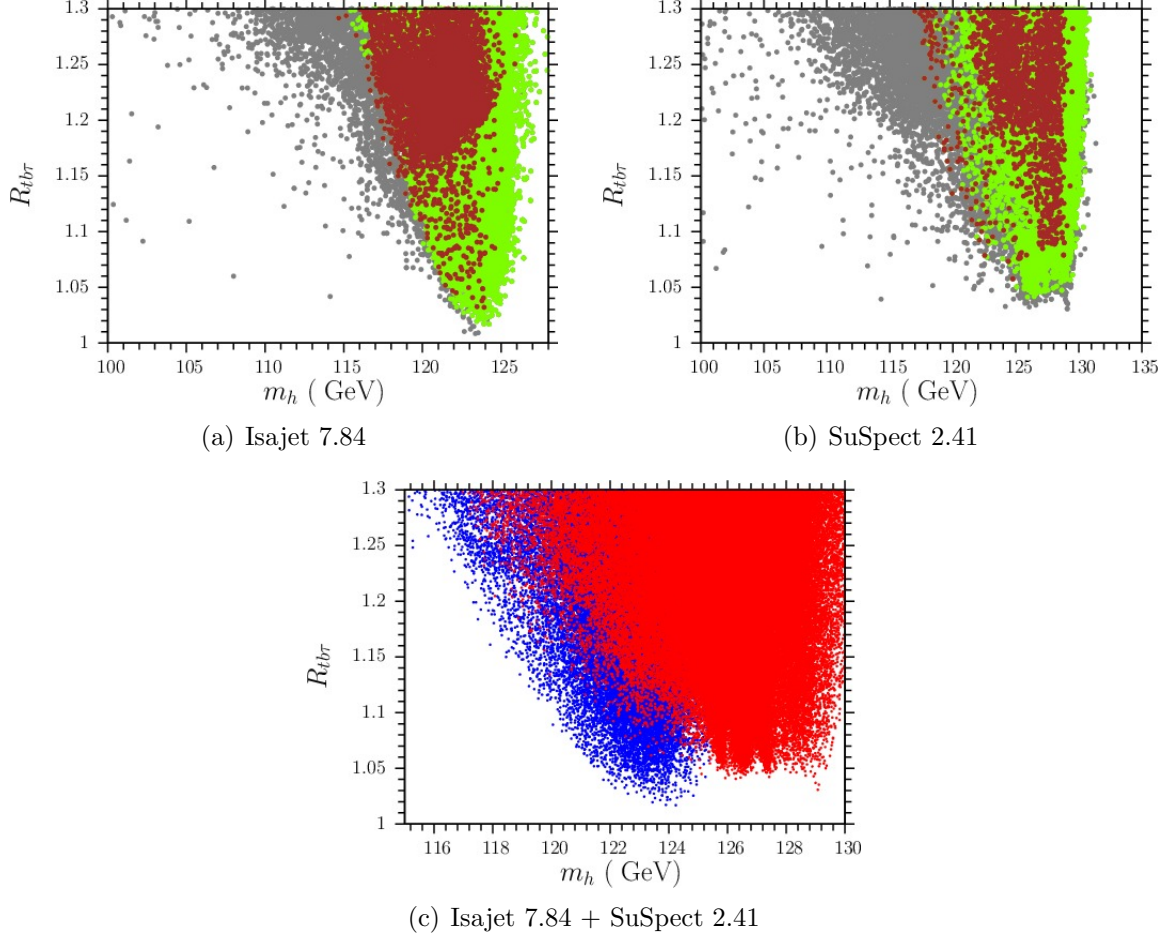


Figure 3: Plot in the $R_{t\bar{t}\tau} - m_h$ plane. Panel 3(a) shows results obtained from Isajet, while 3(b) shows results from obtained SuSpect. Gray points in 3(a) and 3(b) are consistent with REWSB and LSP neutralino. Green points form a subset of the gray and satisfy sparticle mass [35] and B-physics constraints described in Section 4. In addition, we require that green points do no worse than the SM in terms of $(g-2)_\mu$. Brown points form a subset of the green points and satisfy $\Omega h^2 \leq 1$. Panel 3(c) shows data collected with Isajet (blue) and SuSpect (red). The red and blue points satisfy the constraints imposed on the green points in panels 3(a) and 3(b).

5 Higgs Mass Prediction

In this section we discuss the implicit relationship between the SUSY threshold corrections to the Yukawa couplings, REWSB and the light CP-even Higgs mass. We begin with the SUSY threshold corrections to the top, bottom and tau Yukawa couplings which play a crucial role in t - b - τ Yukawa coupling unification. In general, the bottom Yukawa coupling y_b can receive large threshold corrections, while the threshold corrections to y_t are typically smaller [41]. The scale at which Yukawa coupling unification occurs is identified with M_{GUT} , which is the scale of gauge coupling unification. Consider first the unification between $y_t(M_{\text{GUT}})$ and $y_\tau(M_{\text{GUT}})$. The SUSY correction to the tau lepton mass is given by $\delta m_\tau = v \cos \beta \delta y_\tau$. For the large $\tan \beta$ values of interest here, there is sufficient freedom in the choice of δy_τ to achieve $y_t \approx y_\tau$ at M_{GUT} . This freedom stems from the fact that $\cos \beta \simeq 1/\tan \beta$ for large $\tan \beta$, and so we may choose an appropriate δy_τ and $\tan \beta$ to give us both the correct τ lepton mass and $y_t \approx y_\tau$ at M_{GUT} . The SUSY contribution to δy_b has to be carefully monitored in order to achieve Yukawa coupling unification $y_t(M_{\text{GUT}}) \approx y_b(M_{\text{GUT}}) \approx y_\tau(M_{\text{GUT}})$.

We choose the sign of δy_i ($i = t, b, \tau$) from the perspective of evolving y_i from M_{GUT} to M_Z . With this choice, δy_b must receive a negative contribution ($-0.27 \lesssim \delta y_b/y_b \lesssim -0.15$) in order to realize Yukawa coupling unification. The leading contribution to δy_b arises from the finite gluino and chargino loop corrections, and in our sign convention, it is approximately given by [41]

$$\delta y_b^{\text{finite}} \approx \frac{g_3^2}{12\pi^2} \frac{\mu m_{\tilde{g}} \tan \beta}{m_{\tilde{b}}^2} + \frac{y_t^2}{32\pi^2} \frac{\mu A_t \tan \beta}{m_{\tilde{t}}^2}, \quad (10)$$

where g_3 is the strong gauge coupling, $m_{\tilde{g}}$ is the gluino mass, $m_{\tilde{b}}$ ($m_{\tilde{t}}$) is the heaviest sbottom (stop) mass, and A_t is the top trilinear SSB coupling. The logarithmic corrections to y_b are positive, which leaves the finite corrections to provide for the correct overall negative δy_b in order to realize YU.

For models with gaugino mass unification or same sign gauginos, the gluino contribution (first term in Eq.(10)) is positive for $\mu > 0$. Thus, the chargino contribution (second term in Eq.(10)) must play an essential role in providing the required negative contribution to δy_b . This can be achieved [42, 44] only for

$$m_{16} \gg M_{1/2}, \quad m_{16} \gtrsim 6 \text{ TeV} \quad \text{and} \quad A_0/m_{16} \sim -2.6. \quad (11)$$

One could lower the sparticle mass spectrum by considering opposite sign gaugino SSB terms [3] which is allowed by the 4-2-2 model [43]. In SO(10) GUT, non-universality of SSB gaugino masses with opposite signs can be generated through various SO(10) non singlet representations responsible for SUSY breaking [24].

In particular, for $M_3 < 0$, $M_2 > 0$ and $\mu > 0$, the gluino contribution to δy_b has the correct sign to obtain the required b -quark mass, and furthermore, it is not

necessary to have very strong relations among SSB fundamental parameters. Yukawa coupling unification in this case is achieved for

$$m_{16} \gtrsim 300 \text{ GeV} \quad \text{and} \quad M_3(M_2) \geq m_{16}, \quad \text{as well as for} \quad M_3(M_2) \leq m_{16}, \quad (12)$$

as opposed to the parameter space given in Eq. (11). This enables us to simultaneously satisfy the requirements of t - b - τ YU, neutralino dark matter abundance and constraints from $(g-2)_\mu$, as well as a variety of other bounds. But for the above mentioned cases the relation $m_{H_d}^2 > m_{H_u}^2$ was imposed at M_{GUT} . It is well known that REWSB cannot be realized if we require $m_{H_d}^2 = m_{H_u}^2$ at M_{GUT} and, in addition, demand exact t - b - τ YU (For Quasi YU case [45] one can even have $m_{H_d}^2 = m_{H_u}^2 = m_{16}^2$ at M_{GUT}). Let's briefly review REWSB condition in light of YU.

The REWSB minimization condition at tree level requires that

$$\mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2} \approx -m_{H_u}^2 - \frac{M_Z^2}{2}. \quad (13)$$

Here the approximate equality works well for large values for $\tan \beta$, which is the case for t - b - τ YU. Plugging the tree level expression of the CP-odd Higgs mass $m_A^2 = m_{H_d}^2 + m_{H_u}^2 + 2\mu^2$ in Eq. (13), we obtain the relation

$$m_{H_d}^2 - m_{H_u}^2 \gtrsim M_Z^2 + m_A^2. \quad (14)$$

On the other hand, a semi-analytical expression for $m_{H_d}^2 - m_{H_u}^2$ at M_Z , in terms of GUT scale fundamental parameters, has the following form [46]

$$m_{H_d}^2 - m_{H_u}^2 \approx -0.13m_{16}^2 - 0.26m_{1/2}^2 - 0.04m_{1/2}A_0 - 0.01A^2, \quad (15)$$

which implies that in order to satisfy the condition in Eq. (14), we should require

$$M_{1/2} > m_{16}. \quad (16)$$

This requirement clearly contradicts the t - b - τ YU condition obtained in Eq. (11) if gaugino mass unification is assumed at M_{GUT} . This is the reason why REWSB cannot occur if precise Yukawa coupling unification and $m_{H_d}^2 = m_{H_u}^2$ conditions are imposed at M_{GUT} . On the other hand, the condition from Eq. (16) can be consistent with the condition presented in Eq. (12), with non-universal gaugino masses at M_{GUT} . The overlap of conditions from Eqs. (16) and (12) gives very characteristic relations among m_{16} and gaugino masses.

We present in Figure 1 the results of the scan over the parameter space listed in Eq.(9) in $M_3 - m_{16}$ plane. The light blue points are consistent with REWSB and $\tilde{\chi}_1^0$ LSP. Points in black correspond to 3% or better t - b - τ YU ($R_{tb\tau} \leq 1.03$) for Isajet data. For SuSpect data, the black points correspond to 5% or better t - b - τ YU. In

Section 6 we will discuss the factors that can result in a few percent difference in the YU obtained in Isajet and SuSpect. We see that 3% in Isajet's case (5% in SuSpect's case) or better t - b - τ YU can occur for M_3 slightly heavier than m_{16} , and $M_3 > 2$ TeV. On the other hand, M_3 significantly affects the low scale stop quark mass and an approximate semi-analytic expression for $m_{\tilde{t}_R}^2$ for t - b - τ YU case is as follows:

$$m_{\tilde{t}_R}^2 \approx 0.27m_{16} + 5.3M_3^2 + 0.4M_2^2 + \dots \quad (17)$$

Using Eq. (17) we can obtain a rough lower bound for stop quarks, namely $m_{\tilde{t}_R} \gtrsim 4$ TeV.

Next we discuss how the findings presented in Figure 1 can affect the CP-even Higgs boson mass calculation. For the actual calculation, both Isajet and SuSpect employ a more elaborate calculation procedure. We include the one-loop contributions to the CP-even Higgs boson mass [47]:

$$[m_h^2]_{MSSM} \approx M_Z^2 \cos^2 2\beta \left(1 - \frac{3}{8\pi^2} \frac{m_t^2}{v^2} t \right) + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[t + \frac{1}{2} X_t \right], \quad (18)$$

where

$$v = 174.1 \text{ GeV}, \quad t = \log \left(\frac{M_S^2}{M_t^2} \right), \quad X_t = \frac{2\tilde{A}_t^2}{M_S^2} \left(1 - \frac{\tilde{A}_t^2}{12M_S^2} \right). \quad (19)$$

Also $\tilde{A}_t = A_t - \mu \cot \beta$, where A_t denotes the stop left and stop right soft mixing parameter and $M_S = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$. Note that one loop radiative corrections to the CP-even Higgs mass depend logarithmically on the stop quark mass and linearly on X_t . These two quantities essentially determine the radiative corrections to the CP-even Higgs boson mass. Because of this, it is interesting to present the result from Figure 1 in terms of X_t and M_S .

In Figure 2 we show the results in the $M_S - A_t/M_S$ plane. The color coding is the same as in Figure 1. From the $M_S - A_t/M_S$ plane, we see that the black points lie in the interval $5 \text{ TeV} < M_S < 9 \text{ TeV}$. This interval reduces further if we require better YU. This means that for good YU, following Eq.(18) the lightest CP even Higgs boson should be relatively heavy owing to the logarithmic dependence on M_S .

Since the growth or decay of a logarithmic function is slow, the logarithmic dependence of m_h on M_S nicely explains the shape of colored points in Figure 3(a) and 3(b). Panel 3(a) shows results obtained from Isajet, while 3(b) shows results obtained using Suspect. The gray points in Figs. 3(a) and 3(b) are consistent with REWSB and LSP neutralino. The green points form a subset of the gray points and satisfy the sparticle mass and B-physics constraints described in Section 4. In addition, we require that green points do no worse than the SM in terms of $(g-2)_\mu$. The brown points form a subset of the green points and satisfy $\Omega h^2 \leq 1$.

An intriguing feature of Figures 3(a) and 3(b) is that the minima of the distribution occurs at the Higgs mass value very close to the observed mass of the SM-like Higgs at the LHC. In other words, nearly perfect t - b - τ YU prefers the current favored value of m_h . We can also understand from Figure 2 why the minima in the Figure 3(a) and 3(b) have relatively small widths. In Figure 2, from the $M_S - A_t/M_S$ plane, we see that the ratio A_t/M_S lies in the very small interval $0.3 < A_t/M_S < 0.7$. On the other hand, it is known [48] that the CP even Higgs boson obtains significant contributions from A_t if $A_t/M_S > 1$. We can therefore conclude that there is no significant contribution from the finite corrections to the CP even Higgs boson mass if we have almost perfect YU, and the Higgs mass is mostly generated from the logarithmic corrections. This is why the minima in Figures 3(a) and 3(b) are not wide.

In Figures 3(c) we show the overlap of the data from Isajet 7.84 and SuSpect 2.41. All the points shown are consistent with REWSB, LSP neutralino, and satisfy the sparticle mass and B-physics constraints described in Section 4. The blue (red) points show results obtained from Isajet (SuSpect). We can see that the minima of the data distribution from Isajet predicts a Higgs mass $m_h \sim 124$ GeV, whereas SuSpect predicts $m_h \sim 126$ GeV. For lower values of $R_{tb\tau}$, the results from the two packages overlap around $m_h \approx 125$ GeV. This observation makes the predicted value of Higgs mass close to 125 GeV obtained from t - b - τ YU condition quite reliable.

6 Higgs and Sparticle Spectroscopy From Isajet and SuSpect

In this section we compare the allowed parameter spaces and the sparticle spectroscopy obtained from Isajet and SuSpect. A comparison of the uncertainties in the sparticle spectroscopy of different packages was done in [32], and, in particular, the threshold effects were compared in [33]. The approach used by the two programs is very similar in that both use two loop RGE running for the gauge and Yukawa couplings. But there are some factors that can lead to numerical differences of a few percent. These include the scale at which the sparticles are integrated out of the theory. In Isajet the SSB parameters are extracted from RGE running at their respective mass scales $m_i = m_i(m_i)$, whereas in SuSpect these parameters are extracted at $M_{EWSB}(\equiv \sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}})$. SuSpect uses α_s in the $\overline{\text{DR}}$ scheme, whereas Isajet uses the $\overline{\text{MS}}$ value. Another source of discrepancy can be the use of bottom pole mass by SuSpect $m_b = m_b(M_Z)$, whereas Isajet uses the mass at the SUSY scale $m_b = m_b(M_{SUSY})$. The default guess of the sparticle masses at the beginning of the RG evolution process is different in Isajet and SuSpect. SuSpect assumes that the full set of MSSM RGEs are valid between M_Z and M_{GUT} . This is also true for Isajet, except that it employs one-loop step-beta functions for gauge and Yukawa couplings.

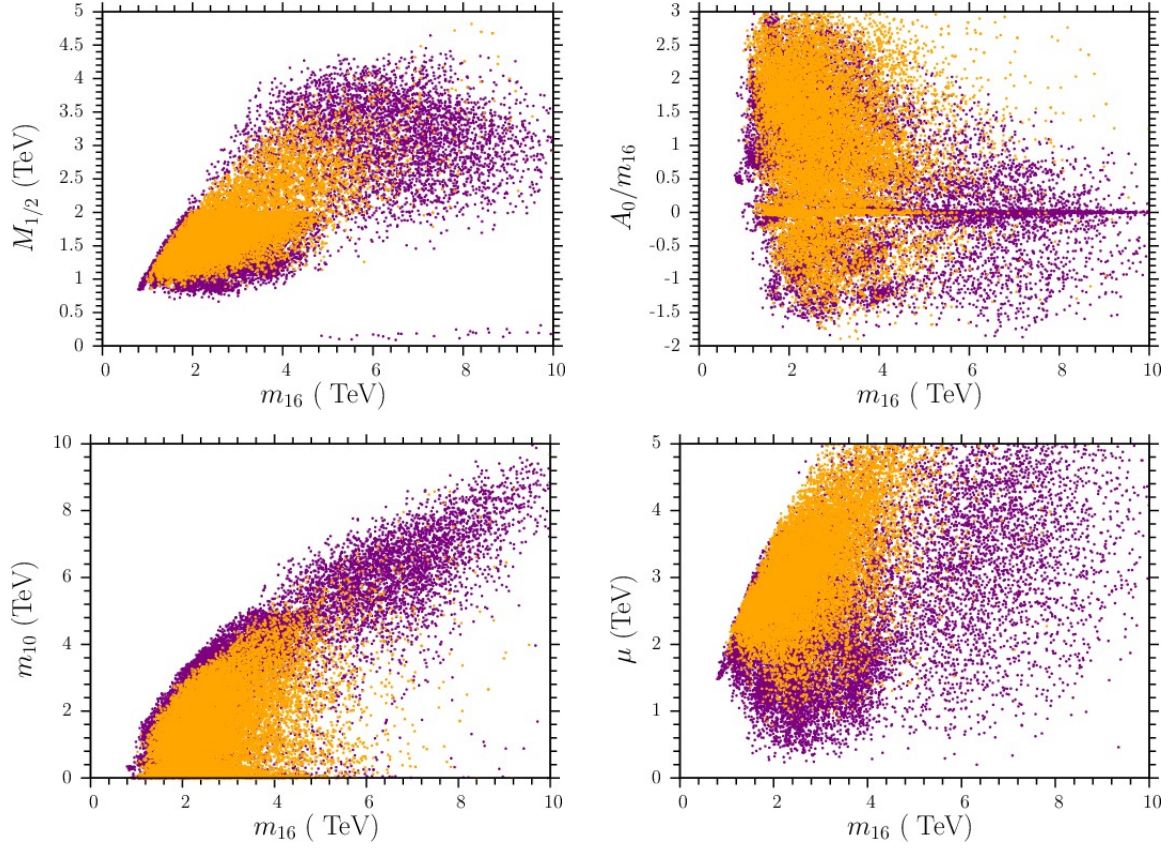


Figure 4: Plots in the $M_{1/2} - m_{16}$, $A_0/m_{16} - m_{16}$, $m_{10} - m_{16}$ and $\mu - m_{16}$ planes. All the points shown are consistent with REWSB, LSP neutralino and satisfy sparticle mass [35] and B- physics constraints described in Section 4. We also require that the points do no worse than the SM in terms of $(g - 2)_\mu$. In addition the satisfy the Higgs mass range $122 \text{ GeV} < m_h < 128 \text{ GeV}$ and $R_{tb\tau} < 1.2$. The purple points show results obtained from SuSpect and yellow points is the data collected using Isajet.

In Figure 4 we show our results in the $M_{1/2} - m_{16}$, $A_0/m_{16} - m_{16}$, $m_{10} - m_{16}$ and $\mu - m_{16}$ planes. All of the points shown are consistent with REWSB, LSP neutralino and satisfy sparticle mass and B-physics constraints described in Section 4. In addition, the points shown satisfy the condition $122 \text{ GeV} < m_h < 128 \text{ GeV}$ and $R_{tb\tau} < 1.2$. The purple points show results obtained from SuSpect, whereas the yellow points correspond to the data collected using Isajet.

We observe that there are some small but notable differences in the results obtained from the two packages. It is interesting to note that for the regions where the points are more dense there is good overlap between the two packages. In Figure 4 under the very dense yellow points there also lie dense purple points. From $M_{1/2} - m_{16}$, $A_0/m_{16} - m_{16}$, $m_{10} - m_{16}$ panels, we see that the best agreement between solutions obtained from Isajet and SuSpect occurs for $m_{16} < 6 \text{ TeV}$, $M_{1/2} < 3 \text{ TeV}$ and $m_{10} < 4 \text{ TeV}$.

In the $\mu - m_{16}$ plane of Figure 4 we see that the solutions from SuSpect with 20% or better t - b - τ YU have lower values of μ compared to the μ values from Isajet. To exemplify this further, in Figure 5 we show a plot in $R_{tb\tau} - \mu$ plane. We see that requiring 20% or better t - b - τ YU allows solutions from SuSpect with $\mu \approx 200 \text{ GeV}$, while requiring the same for Isajet yields $\mu \gtrsim 1 \text{ TeV}$. On the other hand, requiring YU better than 5% leads to similar limits on the values of μ from Isajet and SuSpect, i.e., $\mu \gtrsim 2 \text{ TeV}$. It was noted in [34] that the two codes can differ notably in regions with low μ and $\tan \beta$ values. This difference can stem from the factors previously mentioned, and which may have important implications for natural SUSY.

We observe from Figure 5 that SuSpect does not yield YU better than 5%, with a minimum value of $R_{tb\tau} \sim 1.05$. Isajet, however, predicts even better YU with the minimum value of $R_{tb\tau} \sim 1.02$. This few percent difference could be due to the way threshold effects are evaluated by the two codes [33]. The $R_{tb\tau} - m_{16}$ plane shows that in order to have YU better than 5% the results from Isajet require $600 \text{ GeV} < m_{16} < 2.5 \text{ TeV}$.

In Figure 6 we show results in the $R_{tb\tau} - m_A$, $R_{tb\tau} - m_{\tilde{\tau}_1}$, $R_{tb\tau} - m_{\tilde{t}_1}$, $R_{tb\tau} - m_{\tilde{g}}$, $R_{tb\tau} - m_{\tilde{\chi}_1^0}$ and $R_{tb\tau} - m_{\tilde{\chi}^\pm}$ planes. The color coding is the same as in Figure 3. The data collected with Isajet is used to make the plots in this figure. All panels in Figure 6 indicate that the model predicts relatively narrow ranges for the sparticle masses corresponding to the best t - b - τ YU. The sparticles are heavy enough to evade observation at current LHC energies, but a signal would hopefully be observed during the 14 TeV LHC run.

From the $R_{tb\tau} - m_A$ plane we see that just from the REWSB condition, t - b - τ YU better than 5% predicts that we cannot have a very light CP-odd Higgs boson (A). Similar bounds apply for the heavier CP even H boson and charged H^\pm bosons, since, in the so-called decoupling limit, $m_A^2 \gg M_Z^2$, we have $m_H^2 \simeq m_A^2$ and $m_{H^\pm}^2 \simeq m_A^2 + M_W^2$ [49], where M_W stands for the W -boson mass. The leading decay modes for the heavy

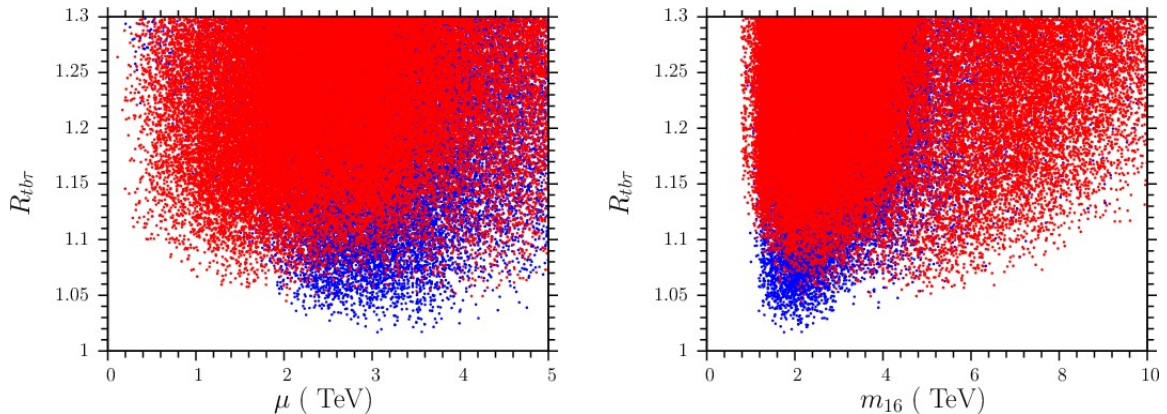


Figure 5: Plots in the $R_{tb\tau} - \mu$, $R_{tb\tau} - m_{16}$ planes. Color coding is the same as in Figure 3(c).

Higgs bosons are $H, A \rightarrow b\bar{b}$ and $H, A \rightarrow \tau^+\tau^-$. The heavy Higgs production cross section and branching ratios depends on $\tan\beta$ and its mass. The MSSM sparticle dependence appears through gluino-squark, Higgsino-squark, bino-sfermion and wino-sfermion loops. The bound on m_A in our scenario is more relaxed since we have non universal gauginos with opposite signs for M_2 and M_3 at the GUT scale. We also find that good t - b - τ YU requires $M_2/M_3 > 1.5$. This will alter the gluino-squark and wino-sfermion loop contributions to the heavy Higgs production cross section and branching ratios [50].

Applying all the collider and B-physics constraints we obtain a lower bound for m_A which is very close to the value corresponding to best YU. Restricting to 5% or better unification and including the constraints presented in Section 4, we obtain the bound $400 \text{ GeV} \lesssim M_A \lesssim 1 \text{ TeV}$. The lower bound is very close to the current experimental limit [10] obtained from the GUT scale gaugino unification condition and can be further tested in near future.

In the $R_{tb\tau} - m_{\tilde{\tau}_1}$ plane we can observe that the preferred values for the stau lepton mass from the point of view of good YU ($R_{tb\tau} < 1.05$) is in the interval $500 \text{ GeV} \lesssim m_{\tilde{\tau}_1} \lesssim 1.5 \text{ TeV}$. The search for a stau in this mass range is challenging at the LHC. In this model, the stau is the NLSP, and this can yield the correct relic abundance through neutralino-stau coannihilation.

The lightest colored sparticle in this scenario is one of the stops. The preferred mass as seen from the $R_{tb\tau} - m_{\tilde{t}_1}$ plane, is around 3-4 TeV. For the gluino, the mass according to the $R_{tb\tau} - m_{\tilde{g}}$ plane is around 5-6 TeV. In principle they can be found at the LHC. We can also see from Figure 6 that the lightest neutralino, for $R_{tb\tau} < 1.05$, is around 500 GeV, and the preferred value from the point of view YU is $\sim 700 \text{ GeV}$. The model also predicts the charginos to be heavier than 2 TeV.

Since the pseudoscalar A boson is being searched for at the LHC, we present in

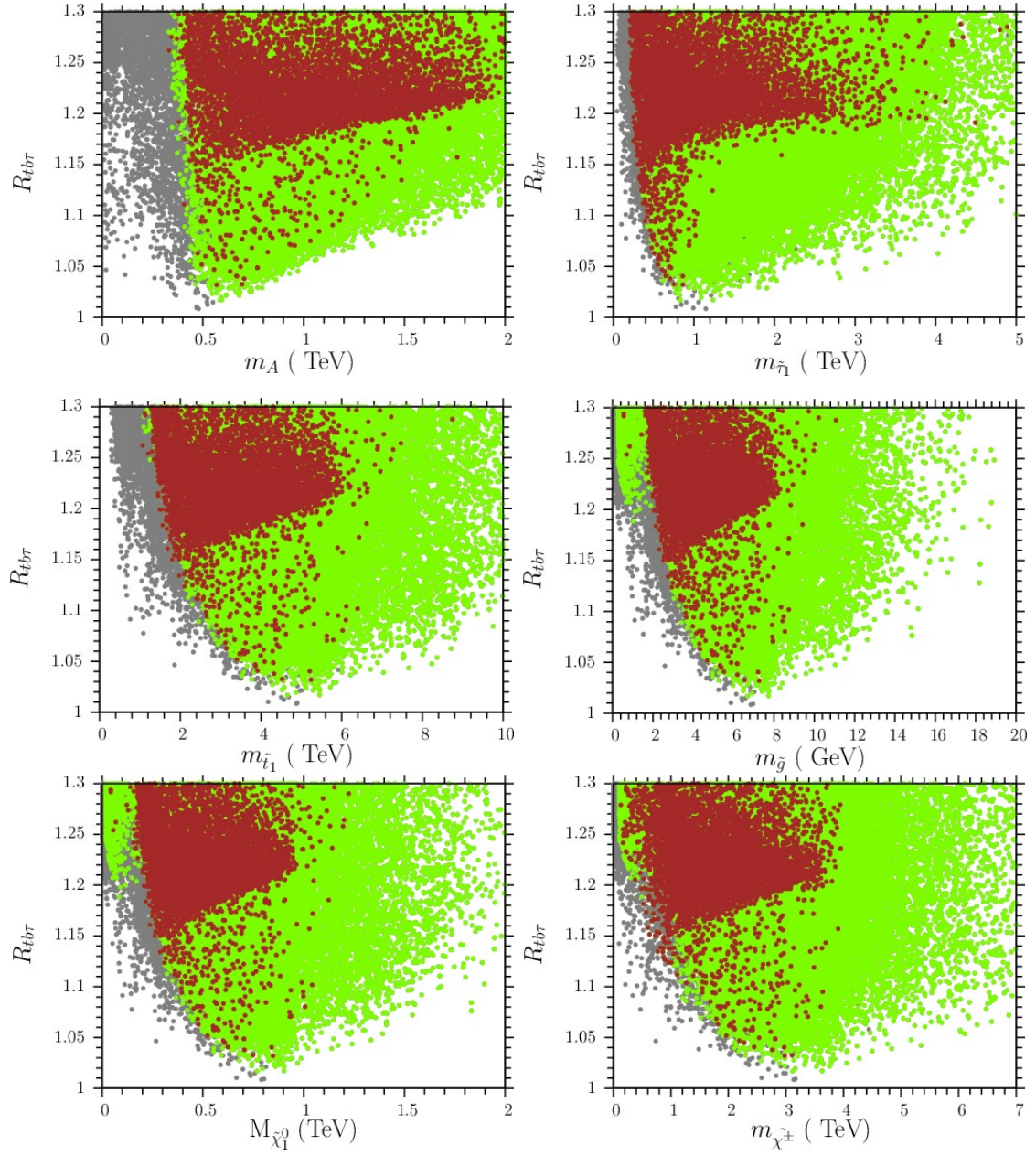


Figure 6: Plot in the $R_{tb\tau} - m_A$, $R_{tb\tau} - m_{\tilde{\tau}_1}$, $R_{tb\tau} - m_{\tilde{t}_1}$, $R_{tb\tau} - m_{\tilde{g}}$, $R_{tb\tau} - m_{\tilde{\chi}_1^0}$ and $R_{tb\tau} - m_{\tilde{\chi}^\pm}$ planes. The color coding is the same as in Figure 3.

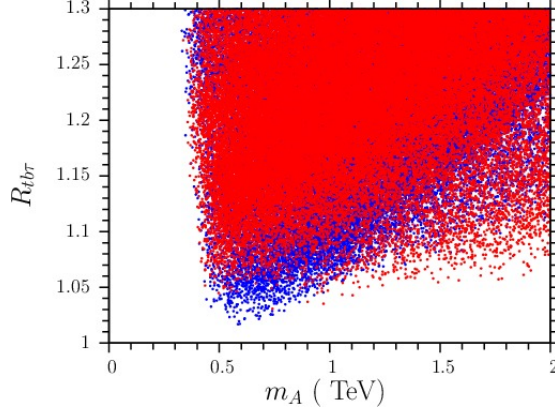


Figure 7: Plot in the $R_{tb\tau} - m_A$ planes. Color coding is the same as in Figure 3(c).

Figure 7 the combined results from Isajet and SuSpect. The color coding is the same as in Figure 6, and we see that the agreement between the two programs is quite satisfying.

In Figure 8 we show the implication of our analysis for direct detection of dark matter. Plots are shown in the $\sigma_{SI} - m_{\tilde{\chi}_1^0}$ and $\sigma_{SD} - m_{\tilde{\chi}_1^0}$ planes. The gray points in the figure are consistent with REWSB and LSP neutralino. The green points form a subset of the gray points and satisfy the constraints described in Section 4. The brown points form a subset of the green points and satisfy $\Omega h^2 \leq 1$, and the orange points form a subset of the brown points with 10% or better YU ($R_{tb\tau} \leq 1.1$). The left panel shows the current and future bounds from CDMS as black (solid and dashed) lines, and as red (solid and dotted) lines for the Xenon experiment. The right panel also shows the current bounds from Super K (solid red line) and IceCube (solid black line), and future reach of IceCube DeepCore (dotted black line).

We can see that the parameter space of this model representing neutralino-stau coannihilation can be tested with these experiments. However models with YU better than 10% yield tiny cross sections which are well below the sensitivity of these experiments. We also observe that good YU predicts a heavy neutralino ($m_{\tilde{\chi}_1^0} \gtrsim 400$ GeV). It is, however, interesting that these orange points also predict a 125 GeV Higgs mass as seen in Figures 3(a) and 3(b). Therefore, t - b - τ YU not only predicts a 125 GeV Higgs but also a relatively heavy dark matter LSP which coannihilates with the stau to yield the correct relic abundance, and also yields tiny cross sections well below the sensitivity of current experiments.

In Table 1 we present four benchmark points with good YU and Higgs mass ~ 125 GeV. The points shown also satisfy the constraints described in Section 4. Points 1 and 2 represent solutions that yield the best YU in Isajet and SuSpect. As described earlier, Isajet yields YU as good as $\sim 2\%$, whereas in SuSpect it is $\sim 5\%$. Point 3 depicts stau coannihilation in addition to a 124 GeV Higgs and $R_{tb\tau} = 1.03$. Point 4 shows that good YU can be attained with the sfermions and Higgs nearly degenerate at M_{GUT} , i.e., $m_{16} \simeq m_{10}$.

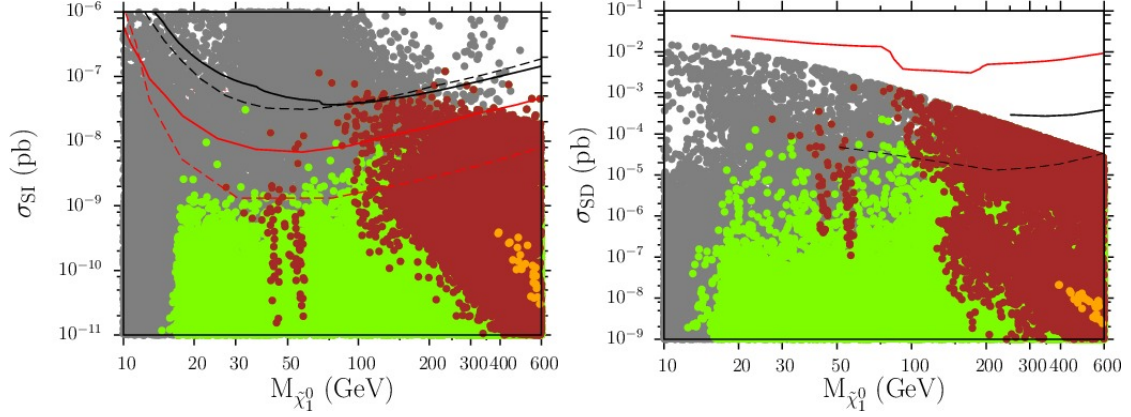


Figure 8: Plots in the $\sigma_{SI} - m_{\tilde{\chi}_1^0}$ and $\sigma_{SD} - m_{\tilde{\chi}_1^0}$ planes. The cross sections are calculated using Isajet. Points shown in gray are consistent with REWSB and LSP neutralino. Green points form a subset of the gray and satisfy sparticle mass [35] and B-physics constraints described in Section 4. In addition, we require that green points do no worse than the SM in terms of $(g - 2)_\mu$. Brown points form a subset of the green points and satisfy $\Omega h^2 \leq 1$. The orange points are a subset of the brown points and satisfy $R_{tb\tau} \leq 1.1$. In the $\sigma_{SI} - m_{\tilde{\chi}_1^0}$ plane, the current and future bounds from the CDMS experiment are represented as black (solid and dashed) lines and as red (solid and dotted) lines for the Xenon experiment. The right panel shows the $\sigma_{SD} - m_{\tilde{\chi}_1^0}$ plane with the current bounds from Super K (solid red line) and IceCube (solid black line) and future reach of IceCube DeepCore (dotted black line).

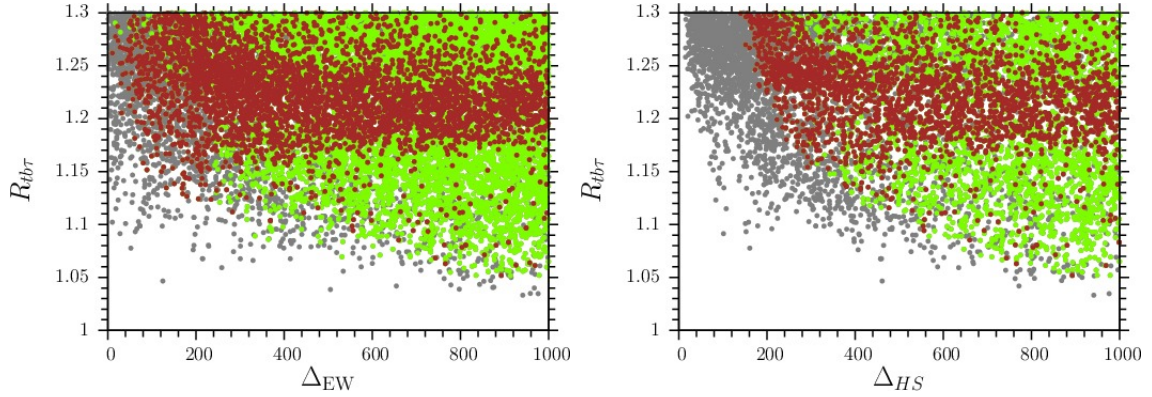


Figure 9: Plots in the $R_{tb\tau} - \Delta_{EW}$ and $R_{tb\tau} - \Delta_{HS}$ planes. Color coding is the same as in Figure 3.

7 Fine tuning constraints for little hierarchy

The latest (7.84) version of ISAJET [29] calculates the fine-tuning conditions related to the little hierarchy problem at Electro Weak (EW) scale and at the GUT scale (HS). We will briefly describe these parameters in this section.

After including the one-loop effective potential contributions to the tree level MSSM Higgs potential, the Z boson mass is given by the following relation:

$$\frac{M_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2. \quad (20)$$

The Σ 's stand for the contributions coming from the one-loop effective potential (For more details see ref. [51]). All parameters in Eq. (20) are defined at EW scale.

In order to measure the EW scale fine-tuning condition associated with the little hierarchy problem, the following definitions are used [51]:

$$C_{H_d} \equiv |m_{H_d}^2/(\tan^2 \beta - 1)|, \quad C_{H_u} \equiv |-m_{H_u}^2 \tan^2 \beta/(\tan^2 \beta - 1)|, \quad C_\mu \equiv |-\mu^2|, \quad (21)$$

with each $C_{\Sigma_{u,d}^{u,d(i)}}$ less than some characteristic value of order M_Z^2 . Here, i labels the SM and supersymmetric particles that contribute to the one-loop Higgs potential. For the fine-tuning condition we have

$$\Delta_{EW} \equiv \max(C_i)/(M_Z^2/2). \quad (22)$$

Note that Eq. (22) defines the fine-tuning condition at EW scale without addressing the question of the origin of the parameters that are involved.

In most SUSY breaking scenarios the parameters in Eq. (20) are defined at a scale higher than M_{EW} . In order to fully address the fine-tuning condition we need to check the relations among the parameters involved in Eq. (20) at high scale. We relate the parameters at low and high scales as follows:

$$m_{H_{u,d}}^2 = m_{H_{u,d}}^2(M_{HS}) + \delta m_{H_{u,d}}^2, \quad \mu^2 = \mu^2(M_{HS}) + \delta \mu^2. \quad (23)$$

Here $m_{H_{u,d}}^2(M_{HS})$ and $\mu^2(M_{HS})$ are the corresponding parameters renormalized at the high scale, and $\delta m_{H_{u,d}}^2$, $\delta \mu^2$ measure how the given parameter is changed due to renormalization group evolution (RGE). Eq. (20) can be re-expressed in the form

$$\begin{aligned} \frac{m_Z^2}{2} &= \frac{(m_{H_d}^2(M_{HS}) + \delta m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2(M_{HS}) + \delta m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} \\ &- (\mu^2(M_{HS}) + \delta \mu^2). \end{aligned} \quad (24)$$

Following ref. [51], we introduce the parameters:

$$\begin{aligned} B_{H_d} &\equiv |m_{H_d}^2(M_{HS})/(\tan^2 \beta - 1)|, \quad B_{\delta H_d} \equiv |\delta m_{H_d}^2/(\tan^2 \beta - 1)|, \\ B_{H_u} &\equiv |-m_{H_u}^2(M_{HS}) \tan^2 \beta/(\tan^2 \beta - 1)|, \quad B_\mu \equiv |\mu^2(M_{HS})|, \\ B_{\delta H_u} &\equiv |-\delta m_{H_u}^2 \tan^2 \beta/(\tan^2 \beta - 1)|, \quad B_{\delta \mu} \equiv |\delta \mu^2|, \end{aligned} \quad (25)$$

and the high scale fine-tuning measure Δ_{HS} is defined to be

$$\Delta_{\text{HS}} \equiv \max(B_i)/(M_Z^2/2). \quad (26)$$

The current experimental bound on the chargino mass ($m_{\widetilde{W}} > 103$ GeV) [35] indicates that either Δ_{EW} or Δ_{HS} cannot be less than 1. The quantities Δ_{EW} and Δ_{HS} measure the sensitivity of the Z-boson mass to the parameters defined in Eqs. (21) and (25), such that $(100/\Delta_{EW})\%$ ($(100/\Delta_{HS})\%$) is the degree of fine-tuning at the corresponding scale.

Based on the definition of high and low scale fine tuning described above we show results in the $R_{tb\tau} - \Delta_{EW}$ and $R_{tb\tau} - \Delta_{HS}$ planes (using Isajet) in Figure 9. We see that the low scale little hierarchy problem becomes more severe for the t - b - τ YU case compared to what we have in the constrained MSSM (CMSSM), but high scale fine tuning is at the same level as in the CMSSM [52]. For t - b - τ YU of around 5%, the EW fine tuning parameter $\Delta_{EW} \sim 800$ and the HS fine tuning parameter is also $\Delta_{HS} \sim 800$. As mentioned above, the fine tuning condition has to be scale invariant which means that cancellation between parameters at a particular scale cannot be more severe compared to same conditions at another scale. Based on this assumption the little hierarchy problem in this model remains the same as we have when gaugino universality is assumed in the theory [51, 52].

8 Conclusion

We have demonstrated how t - b - τ YU is consistent with a 125 GeV Higgs. Our analysis is an extension of the analysis in ref. [6] in several ways. We have highlighted the effects of threshold corrections on the bottom quark Yukawa coupling in this model, and discussed the implicit relationship between these corrections and the Higgs mass. We showed that for YU better than $\sim 5\%$, $M_3 > m_{16}$ at M_{GUT} , with $M_3 \gtrsim 2$ TeV. This, in turn, leads to a heavy stop quark, $m_{\tilde{t}_R} \gtrsim 4$ TeV. The dominant contribution to the Higgs mass arises from the logarithmic dependence of m_h on the stop quark mass. This leads to the prediction $m_h \approx 125$ GeV, consistent with t - b - τ YU better than 5%.

We also compared our results from two different packages, namely Isajet and SuSpect. We found good agreement between the two codes with only a few percent difference between the calculations. One important difference is that Isajet allows YU better than 2%, whereas SuSpect has, at best, 5% YU. Another notable difference is that SuSpect allows for much smaller values of the Higgs mixing parameter μ . This can have implications for natural SUSY since smaller μ values are preferred in resolving the little hierarchy problem.

The two codes also agree well in their predictions of sparticle masses. We find that insisting on YU better than 5% implies that the sparticles are heavy enough

	Isajet	SuSpect	Isajet	Isajet
m_{10}	4.19×10^2	3.82×10^3	4.49×10^2	1.94×10^3
m_{16}	2.13×10^3	2.69×10^3	1.91×10^3	2.00×10^3
M_1	1.89×10^3	2.00×10^3	1.78×10^3	1.51×10^3
M_2	5.67×10^3	6.00×10^3	5.35×10^3	4.53×10^3
M_3	-3.78×10^3	-4.00×10^3	-3.57×10^3	-3.02×10^3
A_0/m_{16}	2.39	1.37	0.03	1.56
$\tan \beta$	47.18	48.05	47.93	47.46
m_t	174.2	173.1	174.2	173.1
μ	3729	1935	2913	2526
m_h	125	126	124	123
m_H	747	491	572	558
m_A	742	491	568	554
m_{H^\pm}	753	500	580	567
$m_{\tilde{\chi}_{1,2}^0}$	895, 3739	955, 1935	848, 2932	709, 2540
$m_{\tilde{\chi}_{3,4}^0}$	3742, 4822	1936, 5043	2935, 4562	2543, 3849
$m_{\tilde{\chi}_{1,2}^\pm}$	3789, 4774	1934, 5043	2978, 4516	2579, 3809
$m_{\tilde{g}}$	7694	7673	7266	6239
$m_{\tilde{u}_{L,R}}$	7667, 6824	8112, 7245	7219, 6415	6295, 5635
$m_{\tilde{t}_{1,2}}$	5331, 6560	5604, 6839	5239, 6367	4390, 5370
$m_{\tilde{d}_{L,R}}$	7668, 6814	8112, 7236	7220, 6406	6296, 5628
$m_{\tilde{b}_{1,2}}$	5553, 6526	5870, 6870	5434, 6333	4591, 5341
$m_{\tilde{\nu}_{1,2}}$	4148	4590	3870	3487
$m_{\tilde{\nu}_3}$	3898	4234	3641	3243
$m_{\tilde{e}_{L,R}}$	4153, 2234	4590, 2780	3875, 2009	3491, 2068
$m_{\tilde{\tau}_{1,2}}$	1094, 3875	1140, 4235	881, 3620	1061, 3225
$\Delta(g-2)_\mu$	3.11×10^{-11}	3.36×10^{-11}	3.71×10^{-11}	4.97×10^{-11}
$\sigma_{SI}(\text{pb})$	1.59×10^{-11}	1.29×10^{-9}	7.08×10^{-11}	1.00×10^{-10}
$\sigma_{SD}(\text{pb})$	4.69×10^{-10}	1.35×10^{-9}	1.60×10^{-9}	2.89×10^{-9}
$\Omega_{CDM} h^2$	6.5	2.8	0.8	4.0
$R_{tb\tau}$	1.02	1.05	1.03	1.04

Table 1: Benchmark points with good Yukawa unification. All the masses are in units of GeV. Point 1, 3 and 4 are generated using Isajet 7.84 whereas point 2 is from SuSpect 2.41. Point 1 and 2 demonstrates how a small value of $R_{tb\tau}$ yields a Higgs mass ~ 125 GeV. Point 3 exhibits stau coannihilation and has a small $R_{tb\tau}$ that agrees with $\Omega h^2 < 1$. Point 4 has $m_{16} \simeq m_{10}$ with good YU.

to evade observation at the current LHC energies, but may be observed during the 14 TeV LHC run. Furthermore, we showed that t - b - τ YU predicts a light CP-odd Higgs boson (A). Restricting to 5% and better YU yields the following bound on the pseudoscalar mass, $400 \text{ GeV} \lesssim M_A \lesssim 1 \text{ TeV}$. Similarly, the bounds on other sparticle masses are $m_{\tilde{g}} \gtrsim 4 \text{ TeV}$, $m_{\tilde{\tau}} \gtrsim 500 \text{ GeV}$ and $m_{\tilde{\chi}^\pm} \gtrsim 2 \text{ TeV}$ for YU 5% or better.

Finally, we also tested the implications of YU for direct detection of dark matter. We found that stau-neutralino coannihilation can lead to the correct dark matter relic abundance. Moreover, insisting on YU better than 10% implies a heavy dark matter candidate ($m_{\tilde{\chi}_1^0} \gtrsim 400 \text{ GeV}$). The neutralino-nucleon cross sections are found to be well below the current sensitivity of direct detection experiments.

Acknowledgments

We would like to thank K.S. Babu, Alexander Pukhov, Shabbar Raza and Zurab Tavartkiladze for useful discussions. This work is supported in part by the DOE Grant No. DE-FG02-91ER40626. This work used the Extreme Science and Engineering Discovery Environment (XSEDE), which is supported by the National Science Foundation grant number OCI-1053575.

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